Safety, Reliability and Risk of Structures, Infrastructures and Engineering Systems – Furuta, Frangopol & Shinozuka (eds)
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An Efficient Bayesian Method for taking into account the Modeling Uncertainties in the Evaluation of Structural Reliability

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Abstract

The seismic performance evaluation of existing buildings is characterized by the large amount of uncertainty in the structural modeling parameters. These modeling uncertainties can be classified into two groups; the uncertainty in the mechanical properties of the construction materials and the uncertainty in the structural detailing (a.k.a. *the defects*). This work employs the Bayesian probability framework in order to make robust estimation of the structural reliability and its dispersion (mean and standard deviation) based on a limited number of structural analyses. The resulting probabilistic assessments of the structural performance are compared to the corresponding assessments based on simulation-based reliability methods and extensive structural analyses. In order to evaluate the overall effect of modeling uncertainties on reliability assessment of existing buildings, the structural modeling uncertainties are taken into account together with the uncertainties in the ground motion representation.

1 INTRODUCTION

A significant portion of the total seismic risk in Italy, evaluated in economic terms, comes from the damages endured by the existing buildings. As a result, more recent Italian seismic codes (e.g., OPCM 2006, Min.LL.PP. 2008a, Min.LL.PP. 2008b) pay particular attention to seismic assessment of existing structures, which is distinguished from that of the new construction by lack of information about both the original features and the current state of the structure. In a previous work (?), the authors have implemented a Bayesian probabilistic framework for a case-study existing structure in order to both characterize the uncertainties in the material properties and structural detailing and also to update the structural reliability by employing the results of in-situ tests and inspections. The objective of this work is to implement the results of structural analysis for a small sample of structural modeling realizations in the Bayesian probabilistic framework in order to make robust estimations of the structural reliability. The structural reliability assessments based on the small-sample interval estimates can be also be performed implementing a set of ground motion records in order to take into account the uncertainty in the ground motion representation.

2 METHODOLOGY

The Bayesian framework for inference is used in order to obtain robust estimates for the structural reliability and its standard deviation based on small-sample simulations of structural modeling parameters. The methodology discussed herein offers a simple alternative to large sample simulations.

2.1 The vector of uncertain parameters

It is assumed that the vector $\underline{\theta}$ represents all the uncertain parameters considered in the problem. The vector $\underline{\theta}$ can include the uncertainties in the mechanical properties of the materials, in the structural construction details (a.k.a., *defects*) and in the representation of the ground motion uncertainty. One of the main characteristics of the construction details is that possible deviations from the original configurations are mostly taken into account in those cases leading to undesirable effects. This explains why the uncertainties related to construction details are also referred to as the *structural defects.* If the probability of failure given the set of parameters *beta* is denoted by $P(F|\beta)$, the expected value (or the robust estimate) for the probability of failure given a set of values Y for the structural performance index can be expressed as:

$$E[P(F|D)] = \int_{\Omega} P(F|\beta)p(\beta|D)d\beta \qquad (1)$$

where $p(\beta|D)$ is the posterior probability distribution for the set of parameters β given the data D and Ω is the space of possible values for β. In a similar way, the robust variance for the probability of failure can be calculated as:

$$\sigma^2_{P(F)|D} = E[P(F|D)^2] - E[P(F|D)]^2 \qquad (2)$$

2.2 The characterization of the uncertainties

Three types of uncertainties are considered herein, namely, the uncertainty in the ground motion input, the uncertainty in the material mechanical properties, and the uncertainties in the structural detailing parameters. Table 1 shows the list of ground motion records used in order to take into account the record-to-record variability. A set of 30 ground motion records are chosen from the European strong motion database for soil type B ($400 \leq V_s \leq$ 600 m/s), with moment magnitude between 5.3 to 7.2 and the epicentral distance between 7 and 87km. The parameters identifying the prior probability distributions for the material mechanical properties (concrete strength and the steel yielding force) have been based on the values typical of the post world-war II construction in Italy (**?**) and (**?**). The prior probability distributions for the structural detailing parameters are defined based on qualitative prior information coming from expert judgement (**?**).

2.3 The structural performance index

When only the structural modeling uncertainties are considered, the definition of structural capacity in this work is based on the limit state of severe damage as proposed by the Italian Code. That is, the onset of critical behavior in the first element, characterized by member chord rotations larger than 3/4th of the corresponding ultimate chord rotation capacity. The structural demand is characterized by the intersection of the code-based inelastic design spectrum and the static pushover curve transformed into that of the equivalent SDOF system (**?**). As an index for the global structural performance, the ratio of structural demand to capacity is used. When the ground motion uncertainty together with the modeling uncertainties are taken into account, the structural performance index is characterized based on the concept of *cut-sets* in structural reliability. A structural cut-set is defined as a set of structural components that, once all of them have failed, they can transform the whole structure or part of it into a mechanism. Among the set of all possible cut-sets, the critical cut-set is the one that first forms a mechanism. Therefore, the performance index is taken as the demand to capacity ratio of the strongest component of the weakest cut-set. In the current work, three types of global mechanism are considered: (a) ultimate rotation capacity in a group of columns* (b) formation of soft stories (c) shear failure in a group of columns. The component yield rotation, ultimate rotation and shear capacities are calculated according to the new Italian Unified Code (**?**) and (**?**). It should be noted that the structural performance in both cases signals failure when it is greater than unity and signals no failure when it is less than or equal to unity.

2.4 Closed form solutions for the structural reliability

The structural reliability or the probability of failure in the case of a structure with modeling uncertainties (no uncertainty in the ground motion) can be expressed by a LogNormal CDF as following:

$$P(Y(\underline{\theta}) > y) = 1 - \Phi(\frac{y - \log \eta_Y}{\sigma_{\log Y}}) \qquad (3)$$

Where Y is the structural performance index and η_Y and $\sigma_{\log Y}$ are the median and the standard

*It is assumed that in the cases where (a)both ends of each of the side columns or (b)both ends of all of the mid columns reach their ultimate rotation capacity, a global mechanism will form.

deviation (of the logarithm) for the probability distribution of the structural performance index. Using Bayesian inference, the posterior probability distribution for median and standard deviation based on data Y can be written as (?):

$$p(\eta,\sigma|Y) = k\sigma^{-(n+1)} \exp - \frac{\nu s^2 + n(log(\eta) - \overline{\log Y})^2}{2\sigma^2}$$

$$k = \sqrt{\frac{n}{2\pi}} \left[\Gamma\left(\frac{\nu}{2}\right)\right]^{-1} \left(\frac{\nu s^2}{2}\right)^{\nu/2} \tag{4}$$

where $Y = \{Y_1, \cdots, Y_n\}$ is the vector of n different realizations of the structural performance index, $\nu = n-1$ and $\overline{\log Y} = \sum \log Y/n$. The expected value and the standard deviation for the probability of failure can be calculated from Equations **??** and **??** based on the posterior probability distribution $p(\eta,\sigma|Y)$ in Equation **??**. Otherwise, the best-estimate values for the median and standard deviation can be calculated either as the maximum likelihood pair for the posterior probability distribution function or based on a given (e.g., 84%) confidence contour.

The structural reliability in the presence of modeling uncertainties and uncertainties in the representation of the ground motion can be calculated from the following LogNormal CDF:

$$P(Y(\underline{\theta}) > 1|S_a) = 1 - \Phi\left(\frac{-\log(\eta_{Y|S_a})}{\beta_{UT|S_a}}\right) \tag{5}$$

where $\beta_{UT|S_a} = \sqrt{\sigma^2_{\log Y|S_a} + \sigma^2_{\log U_1} + \sigma^2_{\log U_2}}$, $\eta_{Y|S_a}$ is the median for the probability distribution of the structural performance index and $\beta_{UT|S_a}$ is the standard deviation for the probability distribution of the structural performance index. The terms $\sigma_{\log Y|S_a}$, $\sigma_{\log U_1}$ and $\sigma_{\log U_2}$, represent the effect of the uncertainty in the ground motion representation, the uncertainty in the material properties and the uncertainty in the structural details, respectively.

Suppose that a selection of n ground motion records are used to represent the effect of ground motion uncertainty on the structural performance index. Let $S_{a,i}$ and Y_i represent the spectral acceleration and the performance index for the ground motion record i, respectively. The data pairs (Y, S_a) are gathered by calculating the structural performance measure for the set of n ground motion records applied at the structural model generated by different realizations of material mechanical properties and structural detailing parameters. Assuming that a linear regression model of the form $\log Y_i = \log a S^b_{a,i} + \epsilon \cdot \sigma$ is used to relate the pairs (Y, S_a), where $\theta = (\log a, b)$ is the vector of regression coefficients, ϵ is described by a standard Normal probability distribution and σ is the standard deviation of the regression.

The posterior probability distribution for standard deviation can be calculated as:

$$p(\sigma|Y,S_a) = \Gamma(\frac{\nu}{2})^{-1}(\frac{\nu s^2}{2})^{\frac{\nu}{2}}\sigma^{-(\nu+1)}e^{-\frac{\nu s^2}{2\sigma^2}} \tag{6}$$

where $\nu s^2 = \sum_{i=1}^n (\log Y_i - a - b \log S_{a,i})^2$, $\nu = n-2$ and the coefficients a and b are equal to:

$$a = \frac{\sum \log Y_i \sum \log S^2_{a,i} - \sum \log S_{a,i} \sum \log Y_i \log S_{a,i}}{n \sum \log S^2_{a,i} - (\sum \log S_{a,i})^2}$$

$$b = \frac{n \sum \log Y_i \log S_{a,i} - \sum \log S_{a,i} \sum \log Y_i}{n \sum \log S^2_{a,i} - (\sum \log S_{a,i})^2} \tag{7}$$

The joint posterior probability distribution for the coefficients of the linear regression $\theta = (\log a, b)$ can be calculated as:

$$p(\theta|Y,S_a) = k[1 + \frac{(\theta-\hat{\theta})^T X^T X(\theta - \hat{\theta})}{\nu s^2}]$$

$$k = \frac{\Gamma(n/2)\sqrt{n \sum \log S^2_{a,i} - (\sum \log S_{a,i})^2} s^{-2}}{(n-2)\Gamma(1/2)^2 \Gamma(n/2-1)} \tag{8}$$

which is the bivariate t-distribution. X is a $n \times 2$ matrix whose first column is a vector of ones and its second column is the vector of $\log S_{a,i}$ and θ is the 2×1 vector of regression coefficients $\log a$ and b. The robust estimates for the expected value and the standard deviation of the failure probability can be obtained from Equations **??** and **??** based on the product of the posterior probability distributions $p(\theta|Y, Sa)$ and $p(\sigma|Y, Sa)$ in Equations **??** and **??**, assuming they are independent.

3 NUMERICAL EXAMPLE

The methodology presented in the previous section is applied to an existing structure as a case study.

3.1 Structural Model

As the case-study, an existing school structure located in Avellino, Italy is considered herein. The structure is situated in seismic zone II according to the Italian seismic Code (?). The structure consists of three stories and a semi-embedded story and its foundation lies on soil type B according to Euro Code 8 (?). For the structure in question, the original design notes and graphics have been gathered. The building is constructed in the 1960's and it is designed for gravity loads only, as it is frequently encountered in the post second world war construction. In Figure ??a,

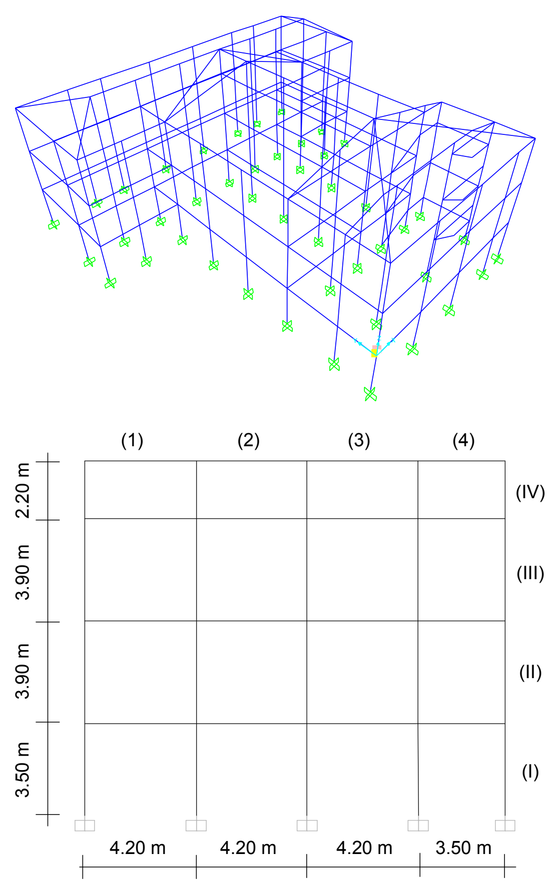

Figure 1: (a) The tri-dimensional view of the scholastic building (b) The central frame of the case-study building

the tri-dimensional view of the structure is illustrated; it can be observed that the building is highly irregular both in plane and elevation. In order to reduce the computational effort, the main central frame in the structure is extracted and used as the structural model (Figure ??b). The columns have rectangular section with the following dimensions: first storey: 40×55 cm^2, second storey: 40×45 cm^2, third storey: 40×40 cm^2, and forth storey: 30×40 cm^2. The beams, also with rectangular section, have the following dimensions: 40×70 cm^2 at first and second storey, and 30×50 cm^2 for the ultimate two floors. It can be inferred from the original design notes that the steel re-bar is of the type Aq40 and the concrete has a minimum resistance equal to 180 kg/cm^2 (?). The finite element model of the frame is constructed assuming that the non-linear behavior in the structure is concentrated in plastic hinges.

3.2 The structural reliability given the design spectrum

The probability distribution for the structural performance index Y (see Section ??), in the presence of uncertainty in the material properties, is calculated in a previous work by the authors (?) using the Monte Carlo simulation with $N_{sim} = 500$ samples as the best estimate. The structural fragility (probability of exceeding a given value of the performance index) is plotted in Figure 2 against the structural performance index Y in thick solid line. The robust structural fragility is calculated from Equation ?? employing the fragility calculated from Equation ?? and joint posterior probability distribution from Equation ??. The data Y used for updating the probability distribution as in Equation ?? has been obtained using Monte Carlo simulation with $n = 7$ samples. The result is plotted in Figure 2 in tiny solid line. In a similar manner, the robust standard deviation for the fragility is calculated from Equation ??; the expected value plus one standard deviation for the fragility curve is plotted in dashed line in Figure 2. It can be observed that with only 7 samples a confidence interval can be constructed for the fragility curve which contains the fragility curve obtained based on Monte Carlo simulation with 500 samples. Figure 3 illustrates the same set of results based on a data set of structural response for $N = 20$ model realizations. It can be observed that the confidence interval based on a sample size of $N = 20$ narrows down significantly with respect to the one based on $N = 7$. The probability distribution for the structural performance index Y in the presence of both uncertainties in the material properties and in the structural detailing parameters is calculated using the Subset Simulation based on $N = 400$ samples (?).

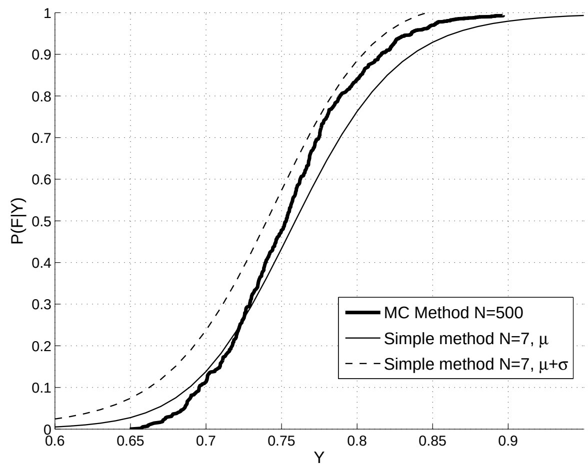

Figure 2: The structural fragility taking into account the uncertainties in the material properties, $N = 7$

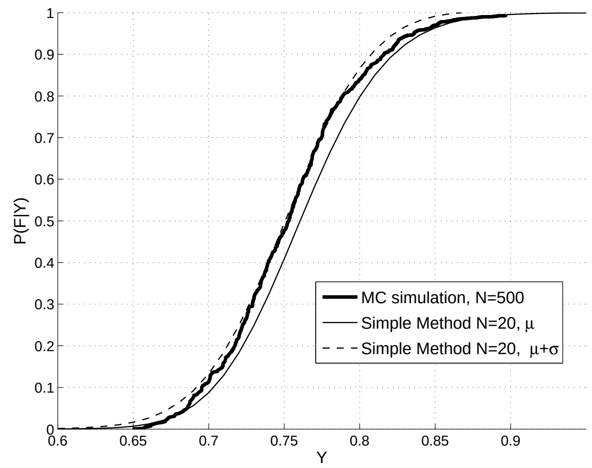

Figure 3: The structural fragility taking into account the uncertainties in the material properties, $N = 20$

The fragility curve derived based on the results of Subset Simulation (the best-estimate) is plotted in Figure 4 in thick solid line. On the other hand, the robust estimate for the fragility curve is again calculated from Equations **??** and **??** employing the posterior probability distribution calculated from Equation **??** based on a data of $N_{sim} = 20$ samples. It should be noted that the $N_{sim} = 20$ samples are generated using Monte Carlo simulation taking into account both uncertainties in the material properties and uncertainties in structural detailing. The robust estimate for the fragility using the simple method (based on small-sample simulation) is plotted in Figure 4 in tiny solid line. The expected value plus standard deviation for the simple method is shown in figure in dashed lines. It can be observed that the confidence interval constructed using the simple method contains the Subset Simulation result.

3.3 The structural reliability taking into account the Ground Motion Uncertainty

The structure is subjected to a set of 30 ground motions in order to consider the record-to-record variability in the ground motion. The structural performance index Y is calculated based on the concept of the critical *cut-set* (**?**) considering both the rotation and the shear capacity in the sections. It turns out the shear failure in columns dominates and the regression coefficients are calculated from Equation **??** as $a = 1.1366$ and $b = 0.24$; the mean square root of the sum of the squares is calculated as $s = 0.10$. The joint posterior probability distributions for a and b is obtained from Equation **??** and the posterior probability distribution for the standard deviation is calculated from Equation **??**. The coefficients of regression and their posterior probability distributions are calculated in two cases: (a) based on the set of 30 ground motion records not considering the modeling uncertainties, and (b) based on the set of 30 ground motion but generating realizations of structural model using Monte Carlo simulation taking into account both the uncertainties in the material properties and the uncertainties in the structural detailing. In both cases, the robust estimate and the standard deviation of the structural fragility are calculated from Equations **??** and **??** employing the probability of failure from Equation **??** and the product of the posterior probability distributions for the coefficient of regression and the standard deviation from Equations **??** and **??**, respectively. The expected value for the probability of failure in case (a) is plotted in Figure 5 in tiny solid line. The expected value plus standard deviation for the probability of failure in case (a) is plotted in tiny dashed line. The mean failure probability and the mean plus standard deviation failure

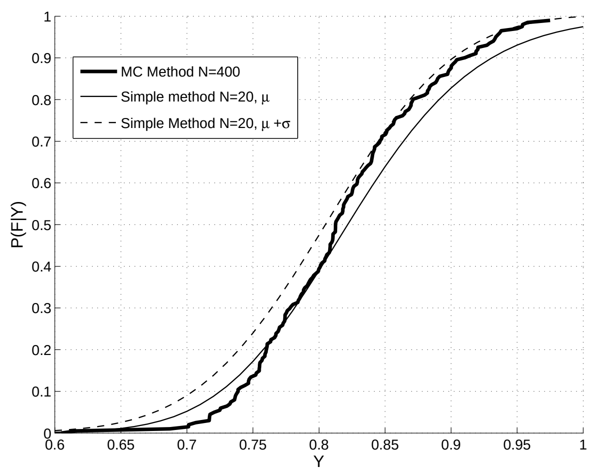

Figure 4: The structural fragility taking into account the uncertainties in material properties and the structural detailing $N = 20$

probability in case (b) are plotted in the same figure with thick solid line and dashed line, respectively. From the confidence intervals obtained for

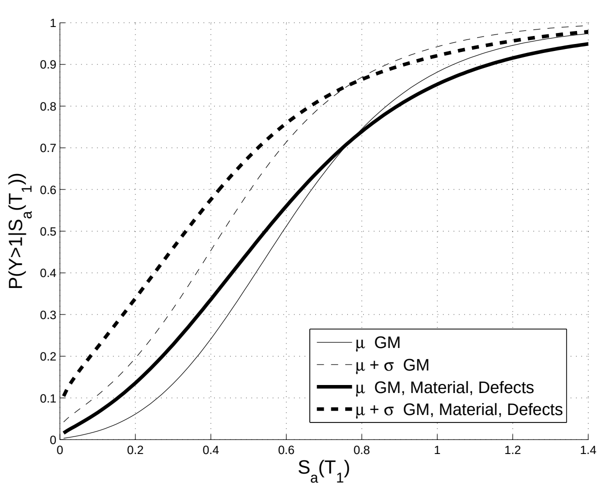

Figure 5: Robust estimates for the failure probability taking into account the record-to-record variability

the failure probability in case (a) and case (b), it can be observed that the presence of structural modeling uncertainties leads to a significant increase in the probability of failure. This confirms that the structural modeling uncertainties can be quite significant in the seismic assessment of existing buildings.

4 CONCLUSIONS

A method is presented for robust interval estimation of the structural reliability taking into account the modeling uncertainties in existing buildings. This method employs small-sample structural analysis results in a Bayesian inference framework leading to posterior probability distributions for median and standard deviation for the structural performance index. The posterior probability distributions for the statistical parameters can be used to obtain a robust confidence interval for the probability of failure. The robust confidence interval is obtained by calculating the posterior (robust) expected value and the standard deviation of the probability of failure based on the posterior probability distribution(s).

The results are presented in two cases, (a) for a specific representation of the ground motion uncertainty, (b) considering the record-to-record variability in ground motion. In case (a) for which the results of extensive simulations were already available, the small-sample methods succeeded in reproducing confidence intervals containing the structural fragility curves based on large-sample simulations. In case (b), the fragility confidence intervals obtained using the small-sample inferences, make it possible to compare the influence of structural modeling uncertainties to that of the ground motion representation. They reconfirm the importance of taking into account the uncertainty both in the material properties and structural detailing in the structural reliability evaluations for an existing building.

In both cases, it is demonstrated that the Bayesian framework for inference is able to provide confidence intervals for the structural reliability, based on small-sample data.

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Table 1: The set of ground motion records; FM is the fault mechanism; ED is the epicentral distance.

Record	M_w	FM	V_{30} (m/s)	ED (km)
Valnerina	5.8	normal	?	23
Friuli, Italy-02	5.9	reverse	412	18
Preveza	5.4	thrust	?	28
Umbria	5.6	normal	546	19
Lazio Abruzzo	5.9	normal	?	36
Etolia	5.3	thrust	405	20
Kyllini	5.9	strike slip	490	14
Irpinia, Italy-01	6.9	normal	600	15
Potenza	5.8	strike slip	494	28
Ano Liosia	6.0	normal	411	20
Adana	6.3	strike slip	?	39
South Iceland	6.5	strike slip	?	15
Patras	5.6	strike slip	665	30
Friuli	6.5	thrust	?	42
Campano Lucano	6.9	normal	472	48
Campano Lucano	6.9	normal	529	16
Kalamata	5.9	normal	486	10
Kalamata	5.9	normal	399	11
Umbria Marche	6	normal	546	11
Umbria Marche	6	normal	450	38
South Iceland	6.5	strike slip	?	7
Duzce 1	7.2	oblique	662	26
Friuli	6.5	thrust	?	87
Campano Lucano	6.9	normal	472	48
Campano Lucano	6.9	normal	529	16
Kalamata	5.9	normal	486	10
Kalamata	5.9	normal	399	11
Umbria Marche	6	normal	546	11
South Iceland	6.5	strike slip	?	7
Duzce 1	7.2	oblique	662	26